EFFECT OF A MOVING WALL ON THE CHARACTERISTICS OF HEAT TRANSFER IN PLANE BOUNDARY LAYERS

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The results of modeling of the hydrodynamics and heat transfer in a boundary layer on a plane constantly moving surface are presented. Specfic features of temperature fields are studied as functions of the boundary conditions and the Prandtl number. Detailed tables of numerical solutions are given.

Introduction. The extensive theoretical and experimental studies of the characteristics of hydrodynamics and heat transfer in boundary layers on plane surfaces are due to the wide range of engineering applications. Hence, the constant interest of specialists in the problem is understandable. This is expressed in the appearance of a vast number of works of a computational and experimental character [1, 2]. However, in spite of the large volume of studies, many important problems have no yet been clarified completely. A case in point is, for example, the effect of a moving wall on the dynamics of temperature fields for different boundary conditions. In analyzing this problem, attention is mainly paid to the hydrodynamic aspects [3-11].

In what follows we present the results of a combined numerical simulation of hydrodynamics and heat transfer on horizontal plane motionless and constantly moving surfaces within the framework of a model of a laminar boundary layer under different temperature boundary conditions within a wide range of Prandtl numbers ($0.001 \le \mathrm{Pr} \le 1000$).

Basic Equations. We introduce the Cartesian coordinates system $x0y$. The flow in the boundary layer is taken to be stationary, laminar, and plane. Then the vector components U, V and the fluid temperature T are only functions of the coordinates x and y. On the surface of a plate we assign the boundary condition of the constancy of temperature T_w (an isothermal surface) or of heat flux q_w (the second-kind boundary condition), and also study specific features of heat transfer in a fluid flow along an adiabatic wall. The actual temperature of a moving plate can be higher or lower than the temperature of surrounding medium T_∞ (or oncoming flow), depending on whether the fluid is heated or cooled by contact with the surface. Then the basic equations describing forced convection on a horizontal surface are written in the form:

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0\,,$$

$$U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial y} = \nu\frac{\partial^2 U}{\partial y^2}\,, \tag{1}$$

$$U\frac{\partial T}{\partial x} + V\frac{\partial T}{\partial y} = \frac{\nu}{\mathrm{Pr}}\frac{\partial^2 T}{\partial y^2}\,.$$

The boundary conditions for the velocity field are

$$y = 0: \quad V = U = 0\,; \quad y \to \infty: \quad U \to U_\infty \quad \text{(motionless surface)}\,; \tag{2}$$

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TABLE 1. Comparison of the Values of $-h'(0, \mathrm{Pr})$ for the Case of an Isothermal Surface

Pr	$U_w = 0, U_\infty \neq 0$			$U_w \neq 0, U_\infty = 0$,
	[12]	[14]	present work	present work
0.0001	0.005588	0.005588	–	–
0.001	0.017316	0.017316	0.017316	–
0.01	0.051588	0.051590	0.051589	0.007990
0.05	–	–	0.105106	0.038269
0.1	0.140029	0.140032	0.140029	0.072863
0.3	–	–	0.214760	0.186256
0.5	–	–	0.259293	0.274960
0.7	–	–	0.292680	0.349236
0.72	–	–	0.295635	0.356084
0.73	–	–	0.297092	0.359474
1	0.332057	0.332058	0.332058	0.443748
2	–	–	0.422308	0.683259
5	–	–	0.576689	1.153872
6.7	–	–	0.636472	1.354449
7	–	–	0.645922	1.387033
10	0.728136	0.728148	0.728141	1.680293
50	–	–	1.247287	3.890918
100	1.57183	1.571855	1.571833	5.544663
500	–	–	2.688271	12.52013
1000	3.38707	3.387096	3.387085	17.74611
10,000	7.29734	7.297423	–	–

$$y = 0: \quad V = 0, \quad U = U_w; \quad y \to \infty: \quad U \to 0 \quad \text{(moving surface)}. \tag{3}$$

Then we write (the temperature field):
an isothermal wall

$$y = 0: \quad T = T_w, \quad y \to \infty: \quad T \to T_\infty; \tag{4}$$

an adiabatic surface

$$y = 0: \quad \frac{\partial T}{\partial y} = 0, \quad y \to \infty: \quad T \to T_\infty; \tag{5}$$

a constant heat flux is assigned on the plate

$$y = 0: \quad -k\frac{\partial T}{\partial y} = q_w, \quad y \to \infty: \quad T \to T_\infty. \tag{6}$$

If the function ΔT is proportional to x^ε, then partial differential equations (1) can be reduced to the system of ordinary differential equations

$$f''' + ff''/2 = 0, \quad \frac{1}{\mathrm{Pr}}h'' + \frac{1}{2}fh' - \varepsilon f'h = 0 \tag{7}$$

by self-similar variables which are determined as follows:

TABLE 2. Comparison of the Values of $1/h(0, \mathrm{Pr})$ for the Case of a Constant Heat Flux on a Wall

Pr	$U_w = 0, U_\infty \neq 0$			$U_w \neq 0, U_\infty = 0$,
	[13]	[14]	present work	present work
0.0001	–	0.008730	–	–
0.001	–	0.026762	0.026762	–
0.01	0.077558	0.077559	0.077558	0.015887
0.05	–	–	0.152949	0.074532
0.1	–	0.200655	0.200654	0.138930
0.3	–	0.301241	0.301239	0.336468
0.5	–	–	0.361003	0.482347
0.7	–	0.405894	0.405894	0.601533
0.72	–	–	0.409871	0.612436
0.73	–	–	0.411832	0.617829
1	0.458970	0.458971	0.458971	0.751177
2	–	–	0.581128	1.126765
3	–	0.666371	–	–
5	–	–	0.791177	1.863085
6.7	–	–	0.872722	2.177220
7	–	0.885620	0.885622	2.228269
10	0.997879	0.997888	0.997883	2.687895
50	–	–	1.707830	6.157147
100	2.15194	2.151968	2.151968	8.754046
500	–	–	3.680130	19.71000
1000	–	4.636736	4.636729	27.91869
10,000	–	9.989653	–	–

$$\psi = (U_* \nu)^{1/2} f(n)\, x^{1/2}, \quad n = (U_* \nu)^{1/2} x^{-1/2} y, \quad \Delta T = T_* h(n)\, x^{\varepsilon}. \tag{8}$$

Note that the power index $\varepsilon = 0$ ($T^* = \Delta T_w$) corresponds to an isothermal wall, $\varepsilon = 1/2$ ($T^* = q_w(\nu/U_*)^{1/2}/k$, to a surface with a constant heat flux, and $\varepsilon = -1/2$, to a linear heat source with power Q on an adiabatic plate ($T_* = Q/C_p \rho (U_* \nu)^{1/2}$). Relations (2) and (3) are transformed to

$$\begin{gathered} f(0) = 0, \quad f'(0) = 0, \quad f'(\infty) = 1 \quad \text{(motionless surface)}; \\ f(0) = 0, \quad f'(0) = 1, \quad f'(\infty) = 0 \quad \text{(moving plate)}. \end{gathered} \tag{9}$$

Similarly, for Eqs. (4)-(6)

$$\begin{aligned} &\text{a)}\ h(0) = 1, \quad h(\infty) = 0, \\ &\text{b)}\ h'(0) = 0, \quad h(\infty) = 0, \\ &\text{c)}\ h'(0) = -1, \quad h(\infty) = 0. \end{aligned} \tag{10}$$

Moreover, in case b) the solution must also satisfy the normalization condition

$$\int_0^\infty f' h\, dn = 1. \tag{11}$$

TABLE 3. Comparison of the Values of $h(0, \Pr)$ for the Case of an Adiabatic Surface

Pr	$U_w = 0, U_\infty \neq 0$		$U_w \neq 0, U_\infty = 0$,
	[15]	present work	present work
0.001	0.01785	0.017847	–
0.01	0.05660	0.056604	0.622135
0.05	–	0.128062	0.635404
0.1	0.18346	0.183468	0.651559
0.3	–	0.331373	0.712010
0.5	–	0.441989	0.767004
0.7	0.53729	0.537291	0.817714
0.72	–	0.546258	0.822584
0.73	–	0.550711	0.825009
1	–	0.664114	0.887500
2	–	1.016264	1.083597
5	–	1.822324	1.511132
6.7	–	2.203827	1.701329
7	2.26722	2.267671	1.732488
10	–	2.863778	2.015207
50	–	8.302557	4.196362
100	13.15523	13.16494	5.843280
500	–	38.46066	12.80976
1000	–	61.04569	18.03368

Results of Calculation. The nonlinear two-point boundary-value problem was solved numerically within the framework of the standard Runge–Kutta scheme by reducing (7), (9)-(11) to the Cauchy problem. The lacking parameters at different values of the Prandtl numbers were determined by the forecast-correlation method and are presented in Tables 1-3. Using the notation of the local coefficient $C_f = \tau_w/\rho U_*^2/2$, we obtain the following complex ($\mathrm{Re}_x = U_* x/\nu$)

$$C_f \mathrm{Re}_x^{1/2} = 2f''(0)\,, \tag{12}$$

which characterizes friction on a wall. Using the notation of the local Nusselt number Nu_x, we introduce the dimensionless temperature complexes

$$\mathrm{Nu}_x \mathrm{Re}_x^{-1/2} = -h'(0, \Pr) \quad \text{(isothermal surface)};$$

$$\mathrm{Nu}_x \mathrm{Re}_x^{-1/2} = 1/h(0, \Pr) \quad \text{(a heat flux } q_w = \text{const is assigned on the wall)}; \tag{13}$$

$$\frac{T_w - T_\infty}{T_*} \mathrm{Re}_x^{1/2} = h(0, \Pr) \quad \text{(adiabatic wall)},$$

which make it easy to determine and estimate the effect of different parameters and factors on the rate of heat transfer.

Since in the case of the boundary conditions (2) $f''(0) = 0.332059$, and for version (3) $f''(0) = -0.443748$ (the sign "minus" denotes that the plane surface possesses propulsion), the coefficient $C_f \mathrm{Re}_x^{1/2}$ for the boundary layer on a constantly moving surface exceeds the value of the same quantity for the boundary layer on a motionless surface by 34%. A similar picture is also observed for the temperature fields: heat transfer increases with the Prandtl number Pr (Pr > 0.5). The sharpness of this increase is enhanced on passing to large Prandtl numbers.

This is due to the fact that at Pr >> 1 the following asymptotic estimates of the change in the quantities $-h'(0, \mathrm{Pr})$ and $1/h(0, \mathrm{Pr})$ are valid:
a moving surface

$$-h'(0, \mathrm{Pr}) = 0.564190\ \mathrm{Pr}^{1/2}, \quad 1/h(0, \mathrm{Pr}) = 0.886227\ \mathrm{Pr}^{1/2}; \tag{14}$$

a motionless wall [14]

$$-h'(0, \mathrm{Pr}) = 0.338720\ \mathrm{Pr}^{1/3}, \quad 1/h(0, \mathrm{Pr}) = 0.46368\ \mathrm{Pr}^{1/3}. \tag{15}$$

Formulas (14) are obtained on the assumption that at a large Prandtl number, when the boundary layer thickness is small compared to the thickness of the hydrodynamic layer, the function $f(n)$ entering into the velocity notation can be replaced by the approximate relation $f(n) = n$ which is valid for those changes of the coordinate n which correspond to the temperature layer region for Pr >> 1. For fluid flow along an adiabatic plate we have

$$\begin{aligned} &U_w = 0, \quad U_\infty \neq 0 \quad h(0, \mathrm{Pr}) = 0.610387\ \mathrm{Pr}^{2/3}; \\ &U_w \neq 0, \quad U_\infty = 0 \quad h(0, \mathrm{Pr}) = 0.56419\ \mathrm{Pr}^{1/2}. \end{aligned} \tag{16}$$

Comparison of the results of numerical and analytical solutions shows that equalities (14)-(16) adequately describe the character and the laws governing thermal processes in plane boundary layers for Pr > 50 (the discrepancy between the exact and approximate values does not exceed 5%).

NOTATION

U, V, longitudinal and transverse velocity components; x, y, longitudinal and transverse coordinates; T, temperature; T_w, T_∞, temperatures of wall and surrounding medium (oncoming flow); ν, k, coefficients of kinematic viscosity and heat conduction; Pr, Prandtl number; U_*, characteristic velocity (U_* for a boundary layer on a motionless surface and U_w for a boundary layer on a constantly moving plate); ρ, density; C_p, heat capacity at constant pressure; $\Delta T = T - T_\infty$, excess temperature.

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